Homework #8. Chapter 27
Magnetic Fields II.

6 • Explain how you would modify Gauss’s law if scientists discovered that single, isolated magnetic poles actually existed.

**Determine the Concept** Gauss’ law for magnetism now reads: “The flux of the magnetic field through any closed surface is equal to zero.” Just like each electric pole has an electric pole strength (an amount of electric charge), each magnetic pole would have a magnetic pole strength (an amount of magnetic charge). Gauss’ law for magnetism would read: ”The flux of the magnetic field through any closed surface is proportional to the total amount of magnetic charge inside.”

20 • A small current element at the origin has a length of 2.0 mm and carries a current of 2.0 A in the +\(z\) direction. Find the magnitude of the magnetic field due to this element and indicate its direction on a diagram at \((a)\) \(x = 2.0\) m, \(y = 4.0\) m, \(z = 0\) and \((b)\) \(x = 2.0\) m, \(y = 0\), \(z = 4.0\) m.

**Picture the Problem** We can substitute for \(I\) and \(d\vec{\ell} \approx \Delta \vec{\ell}\) in the Biot-Savart law 
\[
(d\vec{B} = \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2}),
\]
evaluate \(r\) and \(\hat{r}\) for the given points, and substitute to find \(d\vec{B}\).

Apply the Biot-Savart law to the given current element to obtain:
\[
d\vec{B} = \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2} = \left(10^{-7} \text{ N/A}^2\right) \left(2.0 \text{ A}\right) \left(2.0 \text{ mm}\right) \hat{k} \times \hat{r} = \left(0.400 \text{ nT} \cdot \text{m}^2\right) \hat{k} \times \hat{r}
\]

\((a)\) Find \(r\) and \(\hat{r}\) for the point whose coordinates are \((2.0\ m, 4.0\ m, 0)\):
\[
\hat{r} = (2.0 \text{ m})\hat{i} + (4.0 \text{ m})\hat{j}, \quad r = 2.0\sqrt{5}\ m, \quad \text{and} \quad \hat{r} = \frac{2.0}{2\sqrt{5}}\hat{i} + \frac{4.0}{2\sqrt{5}}\hat{j} = \frac{1.0}{\sqrt{5}}\hat{i} + \frac{2.0}{\sqrt{5}}\hat{j}
\]

Evaluate \(d\vec{B}\) at \((2.0\ m, 4.0\ m, 0)\):
\[
d\vec{B}(2.0\text{m},4.0\text{m},0) = \left(0.400 \text{ nT} \cdot \text{m}^2\right) \frac{\hat{k} \times \left(\frac{1}{\sqrt{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{j}\right)}{2\sqrt{5}\text{ m}} = -\left(18 \text{ pT}\right)\hat{i} + \left(8.9 \text{ pT}\right)\hat{j}
\]
The diagram is shown to the right:

(b) Find \( \mathbf{r} \) and \( \mathbf{r} \hat{r} \) for the point whose coordinates are (2.0 m, 0, 4.0 m):

\[
\mathbf{r} = (2.0 \text{ m}) \hat{i} + (4.0 \text{ m}) \hat{k},
\]

\[
\mathbf{r} = 2.0\sqrt{5} \text{ m},
\]

and

\[
\mathbf{r} = \frac{2.0}{2.0\sqrt{5}} \hat{i} + \frac{4.0}{2.0\sqrt{5}} \hat{k} = \frac{1.0}{\sqrt{5}} \hat{i} + \frac{2.0}{\sqrt{5}} \hat{k}.
\]

Evaluate \( d\mathbf{B} \) at (2.0 m, 0, 4.0 m):

\[
d\mathbf{B}(2.0 \text{ m}, 0, 4.0 \text{ m}) = \left(0.400 \text{ nT} \cdot \text{m}^2\right) \frac{\hat{k} \times \left(\frac{1.0}{\sqrt{5}} \hat{i} + \frac{2.0}{\sqrt{5}} \hat{k}\right)}{(2.0\sqrt{5} \text{ m})^2} = (8.9 \text{ pT}) \hat{f}.
\]

The diagram is shown to the right:

32  ** The current in the wire shown in Figure 27-52 is 8.0 A. Find the magnetic field at point \( P \).

**Picture the Problem** Note that the current segments \( a-b \) and \( e-f \) do not contribute to the magnetic field at point \( P \). The current in the segments \( b-c, c-d, \) and \( d-e \) result in a magnetic field at \( P \) that points into the plane of the paper. Note that the angles \( bPc \) and \( ePd \) are 45° and use the expression for \( B \) due to a straight wire segment to find the contributions to the field at \( P \) of segments \( bc, cd, \) and \( de \).
Express the resultant magnetic field at \( P \):

\[ B = B_{bc} + B_{cd} + B_{de} \]

Express the magnetic field due to a straight line segment:

\[ B = \frac{\mu_0 I}{4\pi R} \left( \sin \theta_1 + \sin \theta_2 \right) \]  \hspace{1cm} (1)

Use equation (1) to express \( B_{bc} \) and \( B_{de} \):

\[ B_{bc} = \frac{\mu_0 I}{4\pi R} (\sin 45^\circ + \sin 0^\circ) \]
\[ = \frac{\mu_0 I}{4\pi R} \sin 45^\circ \]

Use equation (1) to express \( B_{cd} \):

\[ B_{cd} = \frac{\mu_0 I}{4\pi R} (\sin 45^\circ + \sin 45^\circ) \]
\[ = 2 \frac{\mu_0 I}{4\pi R} \sin 45^\circ \]

Substitute to obtain:

\[ B = \frac{\mu_0 I}{4\pi R} \sin 45^\circ + 2 \frac{\mu_0 I}{4\pi R} \sin 45^\circ \]
\[ + \frac{\mu_0 I}{4\pi R} \sin 45^\circ \]
\[ = 4 \frac{\mu_0 I}{4\pi R} \sin 45^\circ \]

Substitute numerical values and evaluate \( B \):

\[ B = 4 \left( 10^{-7} \text{ T} \cdot \text{m/A} \right) \frac{8.0 \text{ A}}{0.010 \text{ m}} \sin 45^\circ \]
\[ = 0.23 \text{ mT into the page} \]

36  ••  An infinitely long wire lies along the \( x \) axis, and carries current \( I \) in the +\( x \) direction. A second infinitely long wire lies along the \( y \) axis, and carries current \( I \) in the +\( y \) direction. At what points in the \( z = 0 \) plane is the resultant magnetic field zero?

**Picture the Problem** Let the numeral 1 denote the current flowing in the positive \( x \) direction and the magnetic field resulting from it and the numeral 2 denote the current flowing in the positive \( y \) direction and the magnetic field resulting from it.

We can express the magnetic field anywhere in the \( xy \) plane using \( B = \frac{\mu_0 2I}{4\pi R} \) and the right-hand rule and then impose the condition that \( \vec{B} = 0 \) to determine the set
of points that satisfy this condition.

Express the resultant magnetic field due to the two current-carrying wires:

\[ \vec{B} = \vec{B}_1 + \vec{B}_2 \]  \hspace{1cm} (1)

Express the magnetic field due to the current flowing in the positive \( x \) direction:

\[ \vec{B}_1 = \frac{\mu_0}{4\pi} \frac{2I_1}{y} \hat{k} \]

Express the magnetic field due to the current flowing in the positive \( y \) direction:

\[ \vec{B}_2 = -\frac{\mu_0}{4\pi} \frac{2I_2}{x} \hat{k} \]

Substitute for \( \vec{B}_1 \) and \( \vec{B}_2 \) in equation (1) and simplify to obtain:

\[ \vec{B} = \frac{\mu_0}{4\pi} \frac{2I}{y} \hat{k} - \frac{\mu_0}{4\pi} \frac{2I}{x} \hat{k} \]

because \( I = I_1 = I_2 \).

For \( \vec{B} = 0 \):

\[ \frac{\mu_0}{4\pi} \frac{2I}{y} - \frac{\mu_0}{4\pi} \frac{2I}{x} = 0 \Rightarrow x = y. \]

\( \vec{B} = 0 \) everywhere on the plane that contains both the \( z \) axis and the line \( y = x \) in the \( z = 0 \) plane.

46 In Figure 27-55, one current is 8.0 A into the page, the other current is 8.0 A out of the page, and each curve is a circular path. (a) Find \( \oint_C \vec{B} \cdot d\vec{l} \) for each path, assuming that each integral is to be evaluated in the counterclockwise direction. (b) Which path, if any, can be used to find the combined magnetic field of these currents?

**Picture the Problem** We can use Ampère’s law, \( \oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_c \), to find the line integral \( \oint_C \vec{B} \cdot d\vec{l} \) for each of the three paths.

(a) Noting that the angle between \( \vec{B} \) and \( d\vec{l} \) is 180°, evaluate \( \oint_C \vec{B} \cdot d\vec{l} \) for \( C_1 \):

\[ \oint_{C_1} \vec{B} \cdot d\vec{l} = -\mu_0 (8.0 \text{ A}) \]

The positive tangential direction on \( C_1 \) is counterclockwise. Therefore, in accord with convention (a right-hand rule), the positive normal direction for the flat surface bounded by \( C_1 \) is out of the page. \( \oint_C \vec{B} \cdot d\vec{l} \) is negative because the
current through the surface is in the negative direction (into the page).

Noting that the net current bounded by \( C_2 \) is zero, evaluate \( \oint_{C_2} \vec{B} \cdot d\vec{l} \):

\[
\oint_{C_2} \vec{B} \cdot d\vec{l} = \mu_0 (8.0 \text{ A} - 8.0 \text{ A}) = 0
\]

Noting that the angle between \( \vec{B} \) and \( d\vec{l} \) is 0°, Evaluate \( \oint_{C_1} \vec{B} \cdot d\vec{l} \) for \( C_3 \):

\[
\oint_{C_1} \vec{B} \cdot d\vec{l} = +\mu_0 (8.0 \text{ A})
\]

(b) None of the paths can be used to find \( \vec{B} \) because the current configuration does not have cylindrical symmetry, which means that \( \vec{B} \) cannot be factored out of the integral.

51  

[SSM] A tightly wound 1000-turn toroid has an inner radius 1.00 cm and an outer radius 2.00 cm, and carries a current of 1.50 A. The toroid is centered at the origin with the centers of the individual turns in the \( z = 0 \) plane. In the \( z = 0 \) plane: (a) What is the magnetic field strength at a distance of 1.10 cm from the origin? (b) What is the magnetic field strength at a distance of 1.50 cm from the origin?

**Picture the Problem** The magnetic field inside a tightly wound toroid is given by \( B = \mu_0 NI/(2\pi r) \), where \( a < r < b \) and \( a \) and \( b \) are the inner and outer radii of the toroid.

Express the magnetic field of a toroid:

\[
B = \frac{\mu_0 NI}{2\pi r}
\]

(a) Substitute numerical values and evaluate \( B \) (1.10 cm):

\[
B(1.10\text{ cm}) = \frac{\left(4\pi \times 10^{-7} \text{ N/A}^2\right) (1000) (1.50 \text{ A})}{2\pi (1.10 \text{ cm})} = 27.3 \text{ mT}
\]

(b) Substitute numerical values and evaluate \( B \) (1.50 cm):

\[
B(1.50\text{ cm}) = \frac{\left(4\pi \times 10^{-7} \text{ N/A}^2\right) (1000) (1.50 \text{ A})}{2\pi (1.50 \text{ cm})} = 20.0 \text{ mT}
\]

82  

The closed loop shown in Figure 27-64 carries a current of 8.0 A in the counterclockwise direction. The radius of the outer arc is 0.60 m and that of the inner arc is 0.40 m. Find the magnetic field at point \( P \).

**Picture the Problem** Let out of the page be the positive \( x \) direction and the numerals 40 and 60 refer to the circular arcs whose radii are 40 cm and 60 cm. Because point \( P \) is on the line connecting the straight segments of the conductor,
these segments do not contribute to the magnetic field at \( P \). Hence the resultant magnetic field at \( P \) will be the sum of the magnetic fields due to the current in the two circular arcs and we can use the expression for the magnetic field at the center of a current loop to find \( \vec{B}_p \).

Express the resultant magnetic field at \( P \):

\[
\vec{B}_p = \vec{B}_{40} + \vec{B}_{60} \quad (1)
\]

Express the magnetic field at the center of a current loop:

\[
B = \frac{\mu_0 I}{2R}
\]

where \( R \) is the radius of the loop.

Express the magnetic field at the center of one-sixth of a current loop:

\[
B = \frac{1}{6} \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{12R}
\]

Express \( \vec{B}_{40} \) and \( \vec{B}_{60} \):

\[
\vec{B}_{40} = -\frac{\mu_0 I}{12R_{40}} \hat{i} \quad \text{and} \quad \vec{B}_{60} = \frac{\mu_0 I}{12R_{60}} \hat{i}
\]

Substitute for \( \vec{B}_{40} \) and \( \vec{B}_{60} \) in equation (1) and simplify to obtain:

\[
\vec{B}_p = -\frac{\mu_0 I}{12R_{40}} \hat{i} + \frac{\mu_0 I}{12R_{60}} \hat{i}
\]

\[
= \frac{\mu_0 I}{12} \left( \frac{1}{R_{60}} - \frac{1}{R_{40}} \right) \hat{i}
\]

Substitute numerical values and evaluate \( \vec{B}_p \):

\[
\vec{B}_p = \left( \frac{4\pi \times 10^{-7} \text{ N/A}^2}{12} \right) \left( \frac{8.0 \text{ A}}{0.60 \text{ m}} - \frac{1}{0.40 \text{ m}} \right) \hat{i} = (-0.70 \mu\text{T}) \hat{i}
\]

or

\[
B_p = 0.70 \mu\text{T} \text{ into the page}
\]

94 ** Figure 27-68 shows a square loop that has 20-cm long sides and is in the \( z = 0 \) plane with its center at the origin. The loop carries a current of 5.0 A. An infinitely long wire that is parallel to the \( x \) axis and carries a current of 10 A intersects the \( z \) axis at \( z = 10 \) cm. The directions of the currents are shown in the figure. (a) Find the net torque on the loop. (b) Find the net force on the loop.

**Picture the Problem** The field \( \vec{B} \) due to the 10-A current is in the \( yz \) plane. The net force on the wires of the square in the \( y \) direction cancel and do not contribute to a net torque or force. We can use \( \vec{r} = \hat{i} \times \vec{F} = \hat{i} \times \hat{F} \), and the expression for the magnetic field due to a long straight wire to express the torque acting on each of the wires and hence, the net torque acting on the loop.
The net torque about the x axis is the sum of the torques due to the forces $\vec{F}_{10}$ and $\vec{F}_{-10}$:

$\vec{\tau}_{\text{net}} = \vec{\tau}_{10} + \vec{\tau}_{-10}$

Substituting for $\vec{\tau}_{10}$ and $\vec{\tau}_{-10}$ yields:

$\vec{\tau}_{\text{net}} = \vec{I}_{10} \times \vec{F}_{10} + \vec{I}_{-10} \times \vec{F}_{-10}$

where the subscripts refer to the positions of the current-carrying wires.

The forces acting on the wires are given by:

$\vec{F}_{10} = (\vec{I}_{10} \times \vec{B}_{10})$

and

$\vec{F}_{-10} = (\vec{I}_{-10} \times \vec{B}_{-10})$

Substitute for $\vec{F}_{10}$ and $\vec{F}_{-10}$ to obtain:

$\vec{\tau}_{\text{net}} = \vec{I}_{10} \times [(\vec{I}_{10} \times \vec{B}_{10})] + \vec{I}_{-10} \times [(\vec{I}_{-10} \times \vec{B}_{-10})]$  \hspace{1cm} (1)

The lever arms for the forces acting on the wires at $y = 10 \text{ cm}$ and $y = -10 \text{ cm}$ are:

$\vec{l}_{10} = (0.10 \text{ m})\hat{j}$ and $\vec{l}_{-10} = -(0.10 \text{ m})\hat{j}$

The magnetic field at the wire at $y = 10 \text{ cm}$ is given by:

$\vec{B}_{10} = \frac{\mu_{0}}{4\pi} \frac{2I}{R \sqrt{2}} \left(-\hat{j} - \hat{k}\right)$

where

$R = \sqrt{(0.10 \text{ m})^2 + (0.10 \text{ m})^2} = 0.141 \text{ m}$.

Substitute numerical values and evaluate $\vec{B}_{10}$:

$\vec{B}_{10} = \frac{4\pi \times 10^{-7} \text{ N/A}^2}{4\pi \sqrt{2}} \frac{2(10 \text{ A})}{0.141 \text{ m}} \left(-\hat{j} - \hat{k}\right) = (10.0 \mu\text{T})\left(-\hat{j} - \hat{k}\right)$
Proceed similarly to obtain: \[ \vec{B}_{-10} = (10.0 \, \vec{\mu T})(-\hat{j} + \hat{k}) \]

Substitute in equation (1) and simplify to obtain:

\[
\vec{\tau}_{net} = (0.10 \, m)\hat{j} \times \left[ (5.0 \, A)(0.20 \, m)\hat{i} \times (10.0 \, \mu T)(-\hat{j} - \hat{k}) \right] \\
- (0.10 \, m)\hat{j} \times \left[ (5.0 \, A)(0.20 \, m)(-\hat{i}) \times (10.0 \, \mu T)(-\hat{j} + \hat{k}) \right] \\
= - (2.0 \, \mu N \cdot m)\hat{k}
\]

\( (b) \) The net force acting on the loop \( \vec{F}_{net} = \vec{F}_{10} + \vec{F}_{-10} \) (2) is the sum of the forces acting on its four sides:

Evaluate \( \vec{F}_{10} \) to obtain:

\[
\vec{F}_{10} = (\vec{I})_{10} \times \vec{B}_{10} = (5.0 \, A)(0.20 \, m)\hat{i} \times (10.0 \, \mu T)(-\hat{j} - \hat{k}) \\
= (10 \, \mu N)\left[\hat{i} \times (\hat{j} + \hat{k})\right] \\
= (10 \, \mu N)(\hat{k} + \hat{j})
\]

Evaluating \( \vec{F}_{-10} \) yields:

\[
\vec{F}_{-10} = (\vec{I})_{-10} \times \vec{B}_{-10} = (5.0 \, A)(-0.20 \, m)\hat{i} \times (10.0 \, \mu T)(\hat{j} + \hat{k}) \\
= (-10 \, \mu N)\left[\hat{i} \times (\hat{j} + \hat{k})\right] \\
= (10 \, \mu N)(\hat{k} + \hat{j})
\]

Substitute for \( \vec{F}_{10} \) and \( \vec{F}_{-10} \) in equation (2) and simplify to obtain:

\[
\vec{F}_{net} = (10 \, \mu N)(\hat{k} + \hat{j}) + (10 \, \mu N)(\hat{k} + \hat{j}) = (20 \, \mu N)\hat{j}
\]